Continuity and Differentiability

Question 1.
The derivative of f(tan x) w.r.t. g(sec x) at $x = \frac{\pi}{4}$, where f'(1) = 2 and g'(√2) = 4, is
(a) $\frac{1}{\sqrt{2}}$
(b) √2
(c) 1
(d) 0
Answer:
(a) $\frac{1}{\sqrt{2}}$

Question 2.

The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

(a) 1 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Answer:
(c) $\frac{2}{3}$

Question 3.

The derivative of

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) \text{ with respect to } \tan^{-1}\left(\frac{2x}{1-x^2}\right) \text{ is}$$

(a) 0 (b) 1

(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

Answer:
(b) 1

Question 4.
img src="https://live.staticflickr.com/65535/50354653758_a00e3fc2ee_o.png" width="374" height="162" alt="Maths MCQs for Class 12 with Answers Chapter 5 Continuity and Differentiability Q34">
Answer:
(c) $\frac{5}{16t^6}$

Question 5.

If $y = (x+\sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$

(c) $-y$ (d) $2x^2y$

Answer:
(a) n^2y

Question 6.

If $x = a\sin\theta$ and $y = b\cos\theta$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{a}{b^2}\sec^2\theta$ (b) $\frac{b}{a}\sec^2\theta$

(c) $\frac{b}{a^2}\sec^3\theta$ (d) $-\frac{b}{a^2}\sec^3\theta$

Answer:
(d) $-\frac{b}{a^2}\sec^3\theta$

Question 7.

If $y = a^x, b^{2x-1}$, then $\frac{d^2y}{dx^2}$ is

(a) $y^2 \cdot \log ab^2$ (b) $y \cdot \log ab^2$

(c) $y \cdot (\log ab^2)^2$ (d) $y \cdot (\log a^2 b)^2$

Answer:
(c) y. $(\log ab^2)^2$

Question 8.

If $y = \frac{\ell n\, x}{x}$, then the value of $y''(e)$ is

(a) 1 (b) $-\frac{1}{e}$

(c) $-\frac{1}{e^2}$ (d) $-\frac{1}{e^3}$

Answer:
(d) $-\frac{1}{e^2}$

Question 9.

If $x = a(\cos\theta + \theta\sin\theta)$, y

$= a(\sin\theta - \theta\cos\theta)$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{\sec^3\theta}{a\theta}$ (b) $\frac{\sec^2\theta}{\theta}$

(c) $a\theta\cos^3\theta$ (d) $\frac{\sec^2\theta}{a}$

Answer:
(a) $\frac{\sec^3\theta}{a\theta}$

Question 10.

If $y^2 = ax^2 + bx + c$, then $\frac{d}{dx}\left(y^3 y_z\right) =$

(a) 1 (b) -1

(c) $\frac{4ac - b^2}{a^2}$ (d) 0

Answer:
(d) 0

Question 11.

If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then the value of $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

(a) $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\frac{\pi}{6}}$

(c) $\frac{1}{\sqrt{6}}$ (d) $\frac{\pi}{\sqrt{6}}$

Answer:
(b) $-\sqrt{\frac{\pi}{6}}$

Question 12.

If $\sqrt{(x+y)} + \sqrt{(y-x)} = a$, then $\frac{dy}{dx} =$

(a) $\frac{\sqrt{(x+y)} - \sqrt{y-x}}{\sqrt{y-x} + \sqrt{x+y}}$ (b) $\frac{2\sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}}$

(c) $\frac{x + y + \sqrt{xy}}{\sqrt{x+y}}$ (d) $\frac{x^2 + y^2 + 2xy}{x^2 + y^2}$

Answer:
(a) $\frac{\sqrt{(x+y)} - \sqrt{y-x}}{\sqrt{y-x} + \sqrt{x+y}}$

Question 13.

If $xy^2 = ax^2 + bxy + y^2$, then find $\frac{dy}{dx}$

(a) $\frac{2ax + by + y^2}{2xy + bx + 2y}$ (b) $\frac{2ax + by - y^2}{2xy - bx - 2y}$

(c) $\frac{ax + by - xy}{xy + x^2 + y^2}$ (d) $\frac{2x^2 + axy + y^2}{x^2 + y^2 + 2xy}$

Answer:
(b) $\frac{2ax+by-y^2}{2xy-bx-2y}$

Question 14.

If $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{\pi}{4}$

(c) 0 (d) 1

Answer:
(d) 1

Question 15.

The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

is

(a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{2\sqrt{1-x^2}}$ (d) x

Answer:
(c) $\frac{1}{2\sqrt{1-x^2}}$

Question 16.

If $f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), 0\le x<\frac{\pi}{2}$, then $f'\left(\frac{\pi}{6}\right)$ is

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Answer:
(d) $\frac{1}{2}$

Question 17.

$\frac{d}{dx}\left\{\text{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)\right\}$ is equal to

(a) $-\frac{2}{1+x^2}, x\neq 0$ (b) $\frac{2(1+x)}{1+x^2}, x\neq 0$

(c) $\frac{2(1-x^2)}{(1+x^2)\lvert 1-x^2\rvert}, x\neq \pm 1, 0$

(d) None of these

Answer:
(c) $\frac{2(1-x^2)}{(1+x^2)\lvert 1-x^2\rvert}, x\neq \pm 1, 0$

Question 18.

If $y=\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right), x>0$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) 0

(c) $\frac{\pi}{2}$ (d) None of these

Answer:
(b) 0

Question 19.

If $y = e^{\frac{1}{2}\log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2}\sec^2 x$ (b) $\sec^2 x$

(c) $\sec x \tan x$ (d) $e^{\frac{1}{2}\log(1+\tan^2 x)}$

Answer:
(c) sec x tan x

Question 20.

If $y = e^{3x+7}$, then the value of $\left.\frac{dy}{dx}\right|_{x=0}$ is

(a) 1 (b) 0
(c) −1 (d) $3e^7$

Answer:
(d) $3e^7$

Question 21.
If $x^2 + y^2 = 1$, then
(a) $yy'' - (2y')^2 + 1 = 0$
(b) $yy'' + (y')^2 + 1 = 0$
(c) $yy'' - (y')^2 - 1 = 0$
(d) $yy'' + (2y')^2 + 1 = 0$
Answer:
(b) $yy'' + (y')^2 + 1 = 0$

Question 22.

If $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$, then $\frac{d^2y}{dx^2}$ is

(a) $-3\sqrt{1-y^2}$ (b) $9y$

(c) $-9y$ (d) $3\sqrt{1-y^2}$

Answer:
(c) -9y

Question 23.
The value of c in Rolle's theorem for the function, f(x) = sin 2x in $[0, \frac{\pi}{2}]$ is
(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{6}$
Answer:
(b) $\frac{\pi}{4}$

Question 24.
The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x$, $x \in [0, \pi]$ is
(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $\frac{3\pi}{4}$
Answer:
(d) $\frac{3\pi}{4}$

Question 25.
A value of c for which the Mean value theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is
(a) $2\log_3 e$
(b) $\frac{1}{2}\log_e 3$
(c) $\log_3 e$
(d) $\log_e 3$
Answer:
(a) $2\log_3 e$

Question 26.
The value of c in mean value theorem for the function f(x) = (x – 3)(x – 6)(x – 9) in [3, 5] is
(a) $6 \pm \sqrt{(13/3)}$
(b) $6 + \sqrt{(13/3)}$
(c) $6 - \sqrt{(13/3)}$
(d) None of these
Answer:
(c) $6 - \sqrt{(13/3)}$

Question 27.
The value of c in Mean value theorem for the function f(x) = x(x – 2), $x \in [1, 2]$ is

(a) $\frac{3}{2}$
(b) $\frac{2}{3}$
(c) $\frac{1}{2}$
(d) $\frac{5}{2}$
Answer:
(a) $\frac{3}{2}$

Question 28.

Let $f(x)=\frac{\ln(1+ax)-\ln(1-bx)}{x}, x\neq 0$. If f(x) is continuous at x = 0, then f(0) =

(a) a – b (b) a + b
(c) b – a (d) ln a + ln b

Answer:
(b) ln a + ln b

Question 29.

If $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2}, & x<0\\ a, & x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$ is continuous at x = 0, then a =

(a) 4 (b) 6
(c) 8 (d) none of these

Answer:
(c) 8

Question 30.

The number of discontinuous functions y(x) on [-2, 2] satisfying $x^2 + y^2 = 4$ is
(a) 0
(b) 1
(c) 2
(d) >2
Answer:
(a) 0

Question 31.

Let $f(x)=\frac{1-\tan x}{4x-\pi}, x \neq \frac{\pi}{4}, x \in \left(0, \frac{\pi}{2}\right)$.

If f(x) is continuous in $\left(0, \frac{\pi}{2}\right)$, then $f\left(\frac{\pi}{4}\right)=$

(a) 1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) -1

Answer:
(c) $-\frac{1}{2}$

Question 32.

If $f(x)=\frac{\sqrt{4+x}-2}{x}, x \neq 0$ be continuous at x = 0, then f(0) =

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 2 (d) $\frac{3}{2}$

Answer:
(b) $\frac{1}{4}$

Question 33.

If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\frac{dy}{dx}=$

(a) $\frac{x+1}{x}$ (b) $\frac{1}{1+x}$

(c) $\frac{-1}{(1+x)^2}$ (d) $\frac{x}{1+x}$

Answer:
(c) $\frac{-1}{(1+x)^2}$

Question 34.
If $y = (1 + x)(1 + x^2)(1 + x^4).....(1 + x^{2n})$, then the value of $\frac{dy}{dx}$ at x = 0 is
(a) 0
(b) -1
(c) 1
(d) None of these
Answer:
(c) 1

Question 35.
If $f(x) = -\sqrt{25 - x^2}$, then $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ is equal to

(a) $\frac{1}{24}$ (b) $\frac{1}{5}$

(c) $-\sqrt{24}$ (d) $\frac{1}{\sqrt{24}}$

Answer:
(d) $\frac{1}{\sqrt{24}}$

Question 36.
If $y = ax^2 + b$, then $\frac{dy}{dx}$ at x = 2 is equal to
(a) 4a
(b) 3a
(c) 2a
(d) None of these
Answer:
(a) 4a

Question 37.

If $\sec\left(\frac{x^2-2x}{x^2+1}\right)=y$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y^2}{x^2}$ (b) $\frac{2y\sqrt{y^2-1}(x^2+x-1)}{(x^2+1)^2}$

(c) $\frac{(x^2+x-1)}{y\left(y^2-1\right)}$ (d) $\frac{x^2-y^2}{x^2+y^2}$

Answer:

(b) $\frac{2y\sqrt{y^2-1}(x^2+x-1)}{(x^2+1)^2}$

Question 38.

If $f(x)=(\log_{\cot x}\tan x)(\log_{\tan x}\cot x)^{-1}+\tan^{-1}\frac{4x}{4-x^2}$, then $f'(2)$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 1 (d) -1

Answer:

(a) $\frac{1}{2}$

Question 39.

If $y=\log_{10}x+\log_e y$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y}{y-1}$ (b) $\frac{y}{x}$

(c) $\frac{\log_{10}e}{x}\left(\frac{y}{y-1}\right)$ (d) None of these

Answer:

(c) $\frac{\log_{10}e}{x}\left(\frac{y}{y-1}\right)$

Question 40.

If $y = \log\left[e^x\left(\frac{x-1}{x+2}\right)^{1/2}\right]$, then $\frac{dy}{dx}$ is equal to

(a) 7 (b) $\frac{3}{x-2}$

(c) $\frac{3}{(x-1)}$ (d) None of these

Answer:
(d) None of these

Question 41.

If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x+y}{xy}$ (b) xy

(c) $\frac{x}{y}$ (d) $\frac{y}{x}$

Answer:
(d) [$latex$] $\frac{y}{x}$ [/latex]

Question 42.
If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3] with $c = (2 + \frac{1}{\sqrt{3}})$, find the value of a and b.
(a) a = 11, b = -6
(b) a = 10, b = 6
(c) a = -11, b = 6
(d) a = 11, b = 6
Answer:
(a) a = 11, b = -6

Question 43.
If $y = (\tan x)^{\sin x}$, then $\frac{dy}{dx}$ is equal to
(a) sec x + cos x
(b) sec x + log tan x
(c) $(\tan x)^{\sin x}$
(d) None of these

Answer:
(d) None of these

Question 44.

If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

(a) $\frac{1+x}{1+\log x}$ (b) $\frac{1-\log x}{1+\log x}$

(c) not defined (d) $\frac{\log x}{(1+\log x)^2}$

Answer:
(d) $\frac{\log x}{(1+\log x)^2}$

Question 45.
The derivative of y = (1 – x)(2 – x) ….. (n – x) at x = 1 is equal to
(a) 0
(b) (-1)(n – 1)!
(c) n! – 1
(d) $(-1)^{n-1}(n-1)!$
Answer:
(b) (-1)(n – 1)!

Question 46.
If $x^y \cdot y^x = 16$, then the value of $\frac{dy}{dx}$ at (2, 2) is
(a) -1
(b) 0
(c) 1
(d) none of these
Answer:
(a) -1

Question 47.

If $y = e^{x+e^{x+e^{x+\ldots \text{ to } \infty}}}$, find $\frac{dy}{dx} =$

(a) $\frac{y^2}{1-y}$ (b) $\frac{y^2}{y-1}$

(c) $\frac{y}{1-y}$ (d) $\frac{-y}{1-y}$

Answer:
(c) $\frac{y}{1-y}$